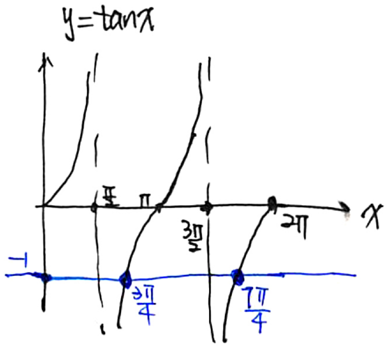
HW 8 #15 #16 Practice Exam #3 (iii)

#15 Find local extrema of $f(x) = \sin x - \cos x$ $[0, 2\pi]$

$$\boxed{\begin{array}{lll} \text{Final answer:} & \frac{3\pi}{4}, 2\pi & \text{local max} \\ & 0, \frac{7\pi}{4} & \text{local min} \end{array}}$$

Work: $f'(x) = \cos x + \sin x$

$f'(x) = 0$ $\sin x = -\cos x$ $\tan x = -1$ $x = \frac{3\pi}{4}, \frac{7\pi}{4}.$



The table

	0	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
f'		$+$		$-$		$+$	
f		↗		↘		↗	
	local min		local max		local min		local max

$\cos\left(\frac{11\pi}{6}\right) + \sin\left(\frac{11\pi}{6}\right)$

$= \cos\left(-\frac{\pi}{6}\right) + \sin\left(-\frac{\pi}{6}\right)$

$= \cos\frac{\pi}{6} - \sin\frac{\pi}{6} = \frac{\sqrt{3}}{2} - \frac{1}{2} > 0$

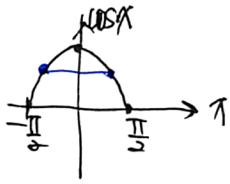
#16 $f(x) = -2x + \tan x \qquad -\frac{\pi}{2} \le x \le \frac{\pi}{2}$ Find local max/min.

$\boxed{\text{Final answer: } -\frac{\pi}{4} \text{ local max} \quad \frac{\pi}{4} \text{ local min}}$

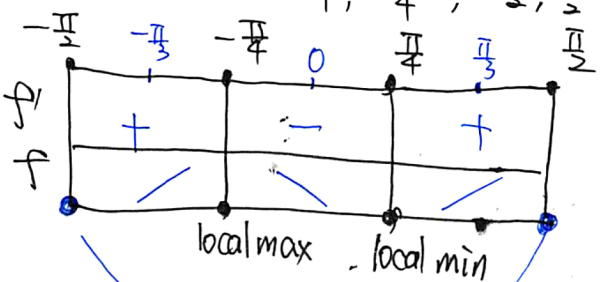
Work: $f'(x) = -2 + \sec^2 x = -2 + \frac{1}{\cos^2 x} = \frac{-2\cos^2 x + 1}{\cos^2 x}$

Critical pts: $-2\cos^2 x + 1 = 0$ or $\cos^2 x = 0$

$\cos x = \frac{\sqrt{2}}{2}$ or $\cos x = -\frac{\sqrt{2}}{2}$ or $\cos x = 0$



$x = \frac{\pi}{4}, -\frac{\pi}{4}, -\frac{\pi}{2}, \frac{\pi}{2}$

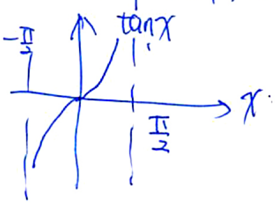


tricky part is that $x = -\frac{\pi}{2}$ may look like local min

$x = \frac{\pi}{2}$ may look like local max.

BUT!!! when $x = -\frac{\pi}{2}, \frac{\pi}{2}$, $\tan x$ is not defined, and so is $f(x)$!

because



Practice Exam #3 (iii)

Find global max / min of $f(t) = 2\cos t + \sin(2t)$ $0 \le t \le \frac{\pi}{2}$

Solutions:

$$f'(t) = -2\sin t + 2\cos(2t)$$

$$= -2\sin t + 2(1-2\sin^2 t)$$

$$= -4\sin^2 t - 2\sin t + 2$$

$$= -2(2\sin t + \sin t - 1)$$

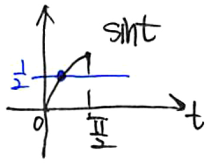
$$= -2(2\sin t - 1)(\sin t + 1) = 0$$

Double angle Formula
$\cos(2t) = \cos^2 t - \sin^2 t$
$= 2\cos^2 t - 1$
$= 1 - 2\sin^2 t$

$\Rightarrow$ $\sin t = \frac{1}{2}$ or $\sin t = -1$

Since $0 \le t \le \frac{\pi}{2}$,

$t = \frac{\pi}{6}$



$[0, \frac{\pi}{2}]$ closed interval $\rightarrow$ great! · closed interval and continuous fuction, we only need to check critical pts and endpts.

$f(0) = 2$

$f(\frac{\pi}{6}) = 2\cos\frac{\pi}{6} + \sin\frac{\pi}{3} = 2 \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} = \frac{3}{2}\sqrt{3}$ $\rightarrow$ global max values.

$\frac{3}{2}\sqrt{3} > 2$, because
$3\sqrt{3} > 4$
$27 = (3\sqrt{3})^2 > 16$

$f(\frac{\pi}{2}) = 2\cos\frac{\pi}{2} + \sin\pi = 0$ $\rightarrow$ global min values

So $(\frac{\pi}{2}, 0)$ is global min